Ex: If $\vec{F}(t) = e^{-t}\hat{i} + \ln(t^2+1)\hat{j} - \tan t\,\hat{k}$

find (a) $\frac{d\vec{F}}{dt}$ (b) $\frac{d^2\vec{F}}{dt^2}$ (c) $\left|\frac{d\vec{F}}{dt}\right|$ (d) $\left|\frac{d^2F}{dt^2}\right|$

at $t = 0$

(a). $\vec{F}(t) = e^{-t}\hat{i} + \ln(t^2+1)\hat{j} - \tan t\,\hat{k}$

$$\frac{d\vec{F}}{dt} = \frac{d}{dt}(e^{-t})\hat{i} + \frac{d}{dt}\{\ln(t^2+1)\}\hat{j} - \frac{d}{dt}(\tan t)\hat{k}$$

$$= -e^{-t}\hat{i} + \frac{2t}{t^2+1}\hat{j} - \sec^2 t\,\hat{k}$$

$$\left.\frac{d\vec{F}}{dt}\right|_{t=0} = -\hat{i} + 0.\hat{j} - 1.\hat{k} = -\hat{i} - \hat{k}$$

(b) $\frac{d^2\vec{F}}{dt^2} = \frac{d}{dt}(-e^{-t})\hat{i} + \frac{d}{dt}\left\{\frac{2t}{t^2+1}\right\}\hat{j} - \frac{d}{dt}(\sec^2 t)\hat{k}$

$$= e^{-t}\hat{i} + \left\{\frac{2}{t^2+1} - \frac{4t^2}{(t^2+1)^2}\right\}\hat{j} - 2\sec^2 t\tan t\,\hat{k}$$

$$\left.\frac{d^2\vec{F}}{dt^2}\right|_{t=0} = \hat{i} + 2\hat{j} - 2.1.0.\hat{k} = \hat{i} + 2\hat{j}$$

(c) $\left|\frac{d\vec{F}}{dt}\right| = \sqrt{(e^{-t})^2 + \left(\frac{2t}{t^2+1}\right)^2 + (-\sec^2 t)^2}$

$$= \sqrt{e^{-2t} + \frac{4t^2}{(t^2+1)^2} + \sec^4 t}$$

$$\left|\frac{d\vec{F}}{dt}\right|_{t=0} = \sqrt{1+0+1} = \sqrt{2}$$

(d) $\left|\frac{dF}{dt^2}\right| = \sqrt{(e^{-t})^2 + \left\{\frac{2}{t^2+1} - \frac{4t^2}{(t^2+1)^2}\right\}^2 + 4\sec^4 t\tan^2 t}$

$$\left|\frac{d^2F}{dt^2}\right|_{t=0} = \sqrt{1+4+4\times 0} = \sqrt{5}$$

Ex: $\vec{F}(t) = (t^2+4t)\hat{i} + (8+et)\hat{j} + (t\sin t)\hat{k}$

Calculate

(i) $\left.\frac{d\vec{F}}{dt}\right|_{t=0}$ (ii) $\left.\frac{d^2\vec{F}}{dt^2}\right|_{t=0}$ (iii) $\left|\left.\frac{d\vec{F}}{dt}\right|_{t=0}\right|$

(iv) $\left|\left.\frac{d^2\vec{F}}{dt^2}\right|_{t=0}\right|$ . . . .